Rotating-polarizer polarimeter for accurate retardance measurement

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We demonstrate an automated polarimeter based on a rotating polarizer for the measurement of linear retardance independent of laser power and detector gain. The retardance is found when a curve is fitted to a unique normalization of the intensity response of the polarimeter over a range of input polarizer orientations. The performance of this polarimeter is optimal for measurements of quarter-wave retardance and minimal for half-wave retardance. Uncertainties are demonstrated by measurements on six stable double Fresnel rhombs of nominal quarter-wave retardance, yielding expanded uncertainties between 0.031° and 0.067°. The accuracy has also been verified by blind comparisons with interferometric and modified null retardance measurement techniques.

1. Introduction

We introduce an automated method for the measurement of retardance with an expanded uncertainty[1] of less than 0.07° on a stable quarter-wave retarder that is a double Fresnel rhomb (see Appendix A and Refs. 2–4). The polarimeter used for this measurement was designed to give a high-precision, high-accuracy measurement of a nominally quarter-wave retarder at 1.32 μm. The performance of the polarimeter is demonstrated by measurements on six separate double-rhomb retarders. Precision is demonstrated by the variance of multiple retardance measurements on each double rhomb, including 68 measurements made on a double rhomb denoted SR6 over a period of 16 months with a standard deviation of only 0.033°. Verification of the accuracy is more difficult. A comparison of retardance measurements made on a stable retarder with this polarimeter, a polarization interferometer,[5] and a modification of the manual null method[6] was carried out in Ref. 7 and is summarized here. Sources of systematic and random errors from the polarimeter and the rhomb are also discussed.

There are many ways to measure linear retardance, including techniques that involve compensators,[8,9] optical heterodyning,[10] electro-optic modulation,[11–14] a rotating sample,[15,16] a rotating retarder,[17] and rotating polarizers.[18] Each technique offers unique advantages and disadvantages such as simplicity, speed of measurement, precision, accuracy, and ability to measure more than just linear retardance. We constructed our polarimeter in response to the need to measure accurately the linear retardance of double-Fresnel-rhomb retarders. In this case, speed is not an issue (our technique currently takes about 20 min to complete a measurement). Our goal was a highly accurate measurement technique whose uncertainties could be rigorously established. We chose our technique because its absolute accuracy is independent of source intensity drift and receiver gain mismatch. Also, our method yields the stated accuracy without the use of compensation techniques that need accurate measurements of the polarimeter's components.

2. Experimental Procedure

Our polarimeter (see Fig. 1) was designed for automated operation with minimal systematic and random uncertainties. The polarimeter has two laser sources: a He–Ne laser at 632.8 nm used for alignment of the system and a multimode laser diode at 1.32 μm (coaligned with the He–Ne) used for the actual polarimetric measurements. After it passes an optical chopper, the light beam passes through a polarizer P_0 and a quarter-wave plate $\lambda/4$ to produce an approximately circular polarization state. A Glan–Thompson calcite polarizer P_1 linearly polarizes the beam before it passes through the retarder under test to a Wollaston beam-splitting polarizer W. The axes of the Wollaston are oriented at ±45° with respect to the polarization axes of the retarder under test. Two germanium pho-
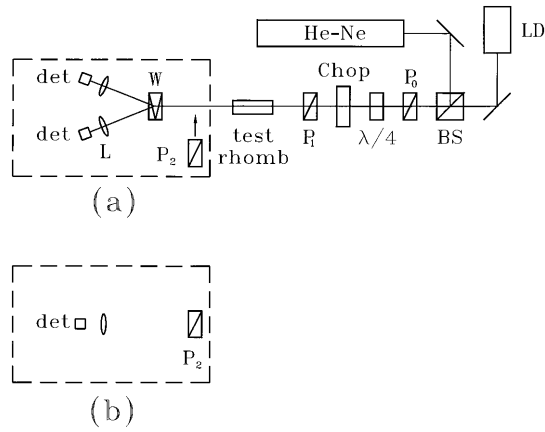
Fig. 1. Design of polarimeter with (a) detection system used for actual retardance measurement (P_2 can be in or out of beam) and (b) detection system for orientation of polarization axes with respect to test retarder's axes.

todiodes detect the orthogonally polarized beams exiting the Wollaston, and the signals are measured with a lock-in amplifier connected to a computer. To measure retardance one could simply orient P_1 at 45° with respect to the polarization axis of the retarder under test, measure the intensities through the two arms of the Wollaston, and calculate the ellipticity of the exiting light.[19] However, our polarimeter uses a modified version of this method that measures 72 data points while P_1 is rotated through 360° in 5° increments. This is done to allow normalization, which removes such sources of error as laser-power fluctuations and detector-gain mismatch.

The initial alignment of the system must be done carefully to avoid systematic errors. The double rhomb being tested was antireflection coated at 1.32 μm, but the He–Ne reflections from the rhomb face can be used to ensure an incident angle of less than 1° from the normal. Another critical alignment is the orientation of the polarizers (P_1 and W) with respect to the polarization eigenaxis of the rhomb. To perform this alignment, the polarimeter is set up with the detection system shown in Fig. 1(b). The polarizer P_1 and analyzer P_2 are held in computer-controlled rotation stages and perform an automated search for a null of the transmitted intensity. At the null orientation, P_1 is parallel to one of the eigenaxes of the rhomb and perpendicular to the axis of P_2. Then P_1 and P_2 are rotated 45°, and the setup is modified to that shown in Fig. 1(a) (with P_2 still in the beam). The Wollaston beam-splitting polarizer W is rotated to give a null for one of its transmitted beams, indicating that the axes of W and P_2 are aligned. Here, P_2 is removed and the system is ready for retardance measurement with P_1 oriented at 45° with respect to the polarization eigenaxes of the double rhomb, and W has its axes parallel and perpendicular to P_1.

The optical setup can be represented by two transfer matrices T_+ and T_-, where the plus and minus

signs indicate which output polarization is detected. In the Jones calculus notation,[20] we find

$$T_+(\delta, \beta, \theta) = \frac{1}{2}\Gamma_+\begin{pmatrix} \cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta \end{pmatrix}$$
$$\times \begin{pmatrix} \cos(\delta/2) & i\sin(\delta/2) \\ i\sin(\delta/2) & \cos(\delta/2) \end{pmatrix}$$
$$\times \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}, \quad (1)$$

and $T_-(\delta, \beta, \theta) = T_+(\delta, \beta + \pi/2, \theta)$ where δ is the retardance of the rhomb, β is the rotational error of W from the desired 45° with respect to the rhomb axis, and θ is the orientation of P_1 with $\theta = 0$ defined as the angle that is 45° away from an eigenaxis of the rhomb. Therefore the intensities measured by the two detectors (one at each output arm of the Wollaston) are

$$I_+(\delta, \beta, \theta) = I_0\Gamma_+{}^2[\sin^2(\delta/2)\cos^2(\theta + \beta)$$
$$+ \cos^2(\delta/2)\sin^2(\theta - \beta)], \quad (2)$$

and

$$I_-(\delta, \beta, \theta) = I_0\Gamma_-{}^2[\sin^2(\delta/2)\sin^2(\theta + \beta)$$
$$+ \cos^2(\delta/2)\cos^2(\theta - \beta)], \quad (3)$$

where I_0 is proportional to the laser power, and Γ_+ and Γ_- are the gains of the two receivers. Taking appropriate ratios of I_+ and I_- yields the normalized quantity

$$I_N(\delta, \beta, \theta) = \left[\frac{I_+(\delta, \beta, \theta)I_-(\delta, \beta, \theta + \pi/2)}{I_-(\delta, \beta, \theta)I_+(\delta, \beta, \theta + \pi/2)}\right]^{1/2}, \quad (4)$$

or

$$I_N(\delta, \beta, \theta)$$
$$= \frac{1}{\cos^2(\beta - \theta)\cos^2(\delta/2) + \sin^2(\beta + \theta)\sin^2(\delta/2)} - 1, \quad (5)$$

which is independent of laser power and detector gain. I_+ and I_- are measured while P_1 is rotated through 360° and I_N is formed as a function of θ. A nonlinear least-squares-fitting algorithm[21] is used to find retardance δ and Wollaston misalignment β as fitting parameters.

Typical measured and fitted data for a quarter-wave double rhomb are shown in Fig. 2. As a reference, an ideal quarter-wave retarder with $\beta = 0$ would yield $I_N(\delta = \pi/2, \beta = 0, \theta) = 1$, independent of θ. As an acceptance criterion for measurements, a χ^2 error coefficient for each data set was calculated as

$$\chi^2 = \sum_i [I_{N,\text{Exp}}(\theta_i) - I_{N,\text{Fit}}(\theta_i)]^2, \quad (6)$$

where $I_{N,\text{Exp}}$ is the experimentally measured ratio, and $I_{N,\text{Fit}}$ is the fit to $I_{N,\text{Exp}}$. χ^2 measures the goodness of fit between Eq. (5) and the measured data. Large values of χ^2 indicate that the model does not
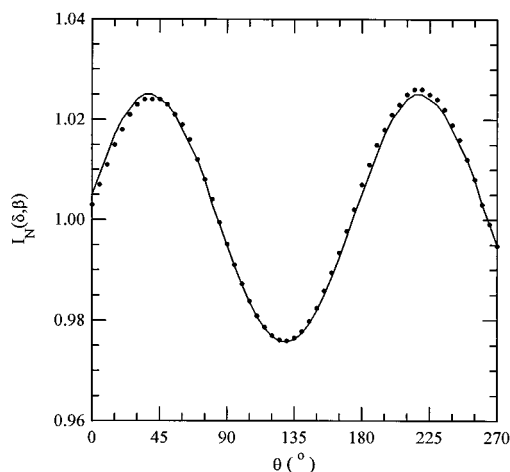
Fig. 2. Typical measurement results of normalized intensity versus polarizer angle. Solid circles represent measured data, solid curve is nonlinear least-squares fit.

adequately describe the experiment (owing to noise, misalignment of the system, perturbation of system during measurement, and so forth). Therefore we rejected data sets that had $\chi^2 > 0.0004$. This threshold was chosen empirically as a value above which the fitted δ were much noisier. As an example of this criterion, we rejected 44 of 112 data sets taken on the double-rhomb SR6. As a test that this χ^2 criterion does not add any systematic bias, we found the value of δ by curve fitting to all 112 data sets and to only the 68 sets that met the χ^2 criterion. The average δ calculated from the 112 data sets and the average δ calculated from the 68 data sets differed by only 0.01°, which is well within the uncertainty of the measurement.

3. Error Evaluation

There are two sources of uncertainty in the retardance measurement: that from the limitations of the measurement equipment and that from the non-ideal nature of the retarder being tested.

A. Computer Simulation

The curve fitting used in the data analysis is a nonlinear process and does not lend itself to simple error analysis. Therefore to estimate uncertainties resulting from various error sources, we used computer simulation. The simulation involved constructing the Jones matrix that represents the measurement system and the retarder under test (with retardance δ_{theory}) and then calculating the normalized transmission I_N [see Eq. (4)] as a function of input polarizer angle θ. This curve was then analyzed with the same software used in the actual measurements. The difference between the fitted value of retardance and δ_{theory} represents the uncertainty of the measurement. To estimate uncertainties resulting from various experimental factors, we modified the Jones matrix of the system and the retarder to simulate actual limitations of the measurement system and

Table 1. Itemized Uncertainty Estimates for Polarimeter

Error Source	Retardance Error[a]	Error Type
Polarizer (extinction ratio)	<0.005°	Systematic/random[b]
Polarizer rotation uncertainty	<0.0001°	Random
Polarizer rotation uncertainty (with 0.8° twist between rhombs)	<0.003°	Random
Beam steering on detector	≪0.24°	Systematic/random[b]
Twist	<0.002°	Systematic
PDL	<0.002°	Systematic

[a]Retardance errors are reported here as maximum worst-case estimates (not standard deviation).
[b]Errors that are due to misalignments caused by polarizer extinction ratios and by beam steering on the detector faces are randomized by realignment of the system.

the rhomb. From the resulting errors, we estimated the uncertainties caused by the various experimental error sources. The results are shown in Table 1.

B. Uncertainties Caused by the Measurement Apparatus

Measurement errors can come from imperfections in the components of the polarimeter. These include the quarter-wave plate, polarizers, detectors, and rotation stages. The effects of these imperfections are discussed below.

1. Wave Plate

The wave plate (labeled λ/4 in Fig. 1) is a true zero-order polymer with approximately 90° retardance. Its purpose is to assure a nominally circular polarization state incident on P_1, which avoids large intensity fluctuations over the course of the measurement. Smaller intensity fluctuations caused by deviation of the wave plate from 90° are removed by the ratio of Eq. (4), so the retardance and stability of the wave plate are not critical.

2. Polarizers

The required performance of the polarizer and the analyzer in this system depends on the value of retardance to be measured. For a quarter-wave retarder, the polarizers need only have an extinction ratio sufficient to locate the eigenaxes of the wave-plate under test accurately. P_1 and P_2 are Glan–Thompson polarizers with extinction ratios better than −55 dB. Determining the eigenaxes of the retarder under test merely by finding the orientation of P_1 and P_2 that gives the best null would be accurate to only ±0.1° for −55 dB polarizers. Instead, the automated system measures transmitted intensity for a range of P_1 and P_2 orientations about the null and fits a second-order polynomial to estimate the null position with an improved two-standard-deviation uncertainty of 0.006°. During the actual retardance measurement, a Wollaston beam-splitting polarizer W with a −50 dB extinction ratio is used. Using the previously mentioned computer simulation, we find that our polarimeter will have an error of less than 0.005° in measured retardance

when polarizers and analyzers of extinction ratios as low as −20 dB are used to measure nominally quarter-wave retarders. We conclude that possible imperfections in all of the polarizers used here have negligible effects on the measured retardance.

The extinction ratio of the polarizers is more important when retardances are measured near zero or half-wave. In that case, the light exiting the rhomb is always nearly linearly polarized, and there are four P_1 orientations for which a near extinction is measured by one of the arms of the Wollaston analyzer. The ability to measure this extinction can be limited by the extinction ratio of the polarizers or the noise floor of the detection system. To make retardance measurements near half-wave, one must use extra care in selecting polarizers and detection electronics. The data analysis routine should also be modified in this case to ignore the data coming from the points of extinction because it will be dominated by noise.

3. Rotation Stages

Our polarimeter was automated with computer-controlled rotation stages to rotate the polarizer P_1 and the analyzer P_2. The rotation stages were driven by stepper motors with a resolution of 0.02°. We consider here the effect of this rotation uncertainty on the measured retardance.

The null position of P_1 and P_2 is determined within a two-standard-deviation accuracy of 0.006°. Therefore the 0.02° stepper-motor resolution is the dominant uncertainty in the position of P_1. The uncertainty in the Wollaston's orientation is ±0.18°, and it comes from the limit imposed by the −50 dB extinction ratio of the Wollaston in finding the null between W and P_2 (no curve fitting was used). Errors owing to these random misalignments were investigated with computer simulation. For retardances between 88° and 92°, the stated uncertainties in the orientations of P_1 and the Wollaston cause random errors in measurement of retardance with a standard deviation of the order of 0.0001°. The simulation was repeated for a 0.8° twist misalignment between the rhombs (to be described in Subsection 3.C.2). Again, the measured retardance had random variations caused by polarizer misalignment, but the standard deviation went up to 0.003°. We conclude that the uncertainties in the orientation of the polarizers have a negligible effect on the measured retardance.

During the actual measurements, we found the fitting parameter β (representing misalignment of the Wollaston polarizer) to have an average value of 0.35° instead of the expected 0. The value varied slightly for different rhombs but always remained positive and nonzero. We are unable to identify the source of this apparent rotation offset. However, we have shown experimentally that it does not affect the measured retardance. Using 112 experimental data sets that we measured for retarder SR6, we performed the fitting routine with and without the β parameter. Fitting to Eq. (5) with β gave χ^2 values lower than fitting without β. However, the mea-

sured value of δ changed by an average of only 0.0005°. Therefore we conclude that this offset of β does not significantly affect our retardance measurements.

4. Detectors

The dual detection system is unaffected by any constant gain mismatch between the two detectors. This is due to normalizing the ratio of detected signals from detectors 1 and 2 to the ratio with the input polarizer rotated by 90° [see Eq. (4)], thus switching the roles of the two detectors and normalizing out any difference in gain.

The major concerns for detector performance are temporal stability and spatial uniformity of the detector responsivity. The temperature dependence of the germanium photodiodes could cause problems resulting from temperature gradients in the laboratory, where the ratio of the gains of the two detectors would not be constant over the course of the measurement. However, the temperature of the laboratory fluctuates randomly over the time scale of multiple measurements, so any errors resulting from incomplete responsivity normalization become random and should be treated statistically.

The spatial uniformity of the detector response becomes an issue when the slight wedge of the polarizer P_1 moves the beam around on the detector surface as P_1 is rotated. The half-cone angle of the measured angular beam deviation by P_1 is 0.04°. We measured the variation in responsivity over the surface of the detector (4 mm × 4 mm active area) to be 2%, so beam steering by P_1 could cause as much as a 2% variation in the amplitude of the detected signal over the course of one rotation of P_1. Through our computer simulation, we have found that a 2% responsivity variation can add a bias to the measured retardance for a single measurement. However, this bias value depends on the responsivity variation over the particular path traced out on the detector surface by the incident laser beam. Simulating multiple measurements with different beam paths gives an average error of 0, with a standard deviation of the measured retardance of approximately 0.08°, or to a 99.7% confidence level, a maximum retardance error of 0.24°.

Moving the detector slightly before each measurement (P_1 scan) would force any beam wander to trace out a different path on the detector surface and to produce agreement with the simulation, thus giving 0 average error. In practice, the detectors were repositioned each time that system was aligned, and a group of retardance measurements was made. Then the system was realigned and another group of measurements was made. Thus each measurement group has different detector-beam positions. Table 2 gives the component of the standard deviation of retardance measurements σ_1 that is due to random errors within these groups and also the component of the standard deviation σ_2 that is due to variation among these groups. There is no significant difference in the standard deviation of these dependent

Table 2. Summary of Measurement Results and Uncertainties on Six Double-Rhomb Retarders

	SR1	SR2[a]	SR2	SR3	SR4	SR5	SR6
Average retardance (°)	89.190	89.392	88.727	89.384	87.963	89.210	90.078
No. of data points	15	10	46	8	9	12	68
No. of groups	1	1	17	2	3	3	28
σ_1 (within group)	0.016	0.033	0.032	0.034	0.015	0.004	0.025
σ_2 (among groups)	NA	NA	—[b]	—[b]	0.017	0.015	0.022
σ_T	0.016	0.033	0.032	0.034	0.022	0.016	0.033
Expanded uncertainty U	0.031	0.067	0.064	0.067	0.045	0.031	0.066

[a]The first SR2 data column includes the measurement results that were used in the comparison between measurement techniques (see Ref. 7). The second column of SR2 data represents a more thorough data set after the rhomb was subjected to extreme humidity that actually changed the retardance (see Ref. 7).

[b]σ_2 for these data sets did not yield a statistically significant difference from 0.

and independent measurements. We conclude that the effects of beam steering are further averaged by other randomizing factors in the experiment (such as temperature fluctuations during the experiment). This conclusion is also supported by the standard deviation σ_T of real measurements being much less than 0.08° (see Table 2).

To experimentally verify that detector surface uniformity added no systematic biases to the measured retardance, we examined the symmetries of the data sets. The intensity ratio [see Eq. (4)] should repeat with a 180° rotation of P_1. (This is seen in Fig. 2.) However, modulation of the intensity ratio owing to beam steering on the detector would have a periodicity of 360°. For each rhomb, we examined every data set and compared the retardance measured, using data taken over a P_1 range 0°–180°, with that measured with data produced in the range 180°–360°. If significant beam-steering errors exist, we would expect the average measured retardance from the 0°–180° range to be significantly different from the average retardance measured for data from the 180°–360° range. For all but one double rhomb, the two data ranges had no statistically significant average difference in their retardances. This indicates that any errors caused by detector spatial nonuniformity were random. Rhomb SR4 was the exception, showing that retardance calculated from the 0°–180° range was consistently larger (approximately 0.07° on average) than that measured with the 180°–360° range. This is evidence of some systematic effect in the measurements, but more testing is necessary to determine if there was actually a bias on the average measured retardance.

We conclude that, with the possible exception of SR4, beam steering and temperature fluctuation of the detector responsivities have only a random effect on the measured retardance with no systematic bias. The simulated random uncertainty due to beam steering ($\sigma = 0.08°$) severely overestimates the standard deviation of actual measurements ($\sim$0.03°). Therefore, in reality, the worst-case beam-steering errors are much less than the previously calculated 3σ estimate of 0.24°.

C. Uncertainties Caused by Test Retarder (Double Fresnel Rhomb)

Nonideal qualities of the test retarder that could affect the performance of the measurement apparatus include surface reflectance, polarization rotation, and polarization-dependent loss (PDL).

1. Surface Reflectance

In a simple retardance measurement, reflections at the surfaces of the retarder allow the probe light to experience multiple passes through the retarder, thus changing the polarization-dependent delay of the emerging light. The error in measured retardance depends on the coherence function of the source, the reflectances of the surfaces of the retarder, and the length of the optical path between those surfaces.[7] Because our source was a multimode laser diode, the envelope of its coherence function likely extends several meters owing to the narrow linewidth of the individual modes. Therefore the effects of multiple reflections must be considered.

It is fortunate that the phase errors resulting from reflections are averaged by the temperature dependence of the optical path length of the double rhomb. The reflection-induced retardance error is a sensitive function of optical path length inside the double rhomb. However, when they are averaged over optical path length, the retardance errors caused by reflection average to 0. This means that a slight temperature change between measurements varies the optical path length of the double rhomb, and that multiple measurements average out reflection errors. For our device, we concluded that a temperature change of 0.3 °C is sufficient to sample all possible phase errors and give a mean phase error of 0 (see Ref. 7). Because the long-term temperature control of the laboratory is within only a few degrees Celsius, we were able to treat reflection errors as random over multiple measurements.

2. Polarization Rotation

The glass used to make the rhombs has no known intrinsic source of optical activity. However, a small

rotation could arise if the two rhombs are glued together slightly twisted with respect to each other. Of the six double rhombs tested, twist misalignments were measured to be between 0.2° and 0.8°. Our computer simulation was used to determine the expected errors caused by such misalignments. Several $I_N(\theta)$ data sets were simulated with a range of twist and retardance values that nominally agreed with the measured values of each of the double rhombs. These I_N data sets were then analyzed. The worst-case difference between the δ_{theory} value used to generate the simulated curve and the value of δ given by the curve fit is considered to be the maximum twist-induced retardance error. We found that this error is independent of the sign of the twist, and it shows a weak dependence on retardance in the vicinity of 90°. The simulation shows that the maximum twist-induced retardance error in any of the tested rhombs is less than 0.002°. This is insignificant in comparison with the random errors of the experiment, and it is not included in the overall uncertainty statement of the polarimeter.

3. Polarization-Dependent Loss

The double rhombs measured in our experiment have no known inherent source of PDL. But, PDL could arise from polarization-dependent Fresnel losses caused by nonnormal incidence reflections from the surfaces of the rhomb. All of the rhombs used in measurements were antireflection coated, with the rhomb-to-air interface giving between 0.02 and 0.8% reflections, and the rhomb-to-rhomb interface giving <0.02% reflections. For a 1° angle of incidence (the worst-case misalignment of our system), polarization-dependent Fresnel reflections could cause a maximum PDL of 8×10^{-5} dB.

We estimated the retardance errors caused by PDL by using our computer simulation. The Jones matrix for the double rhomb was constructed with twice the worst-case PDL at each interface of the rhomb. Performing the simulated measurement for various orientations of the PDL and over a range of expected retardance values from 88° to 92°, we found the error in measured retardance to be always less than 0.002°. This uncertainty is insignificant compared with the random uncertainty of the system and is not included in the overall uncertainty statement of the polarimeter.

4. Systematic Uncertainties

In general, one cannot quantify systematic errors simply by examining the measurement results. To find any systematic errors not yet considered, we compared measurement results on five of the double rhombs with measurements made with two other retardance measurement techniques. The first of these techniques was a polarization interferometer that measured the phase retardance of the rhomb directly.[5] The second technique was a modification of the null method of retardance measurement.[6] Each of these three independent techniques was used to make blind measurements on the five double rhombs. The absolute disagreement between measurements made with this polarimeter and those made with the other two systems was within the estimated uncertainties of the measurement systems and validates the stated accuracy (see Table 2) of this system.[7] This verifies that our uncertainty analysis did not overlook any significant systematic error sources.

Another attempt at identifying systematic errors might be the comparison of measured retardance with a theoretical value determined from the geometry of the rhomb and its index of refraction. However, the retardance of the double rhombs is affected by other less-well-known factors such as degradation of the total-internal-reflection (TIR) surfaces owing to compaction during polishing or adsorption of water vapor. As a result, designing a rhomb of a given retardance is an empirical process that does not lend itself to a sufficiently accurate theoretical prediction of absolute retardance. These issues are discussed in Ref. 7.

5. Random Uncertainties

As described previously in the discussion on detectors, multiple measurements of retardance were made in groups. The system was aligned, and a group of measurements was made. Then the system was realigned and another group of measurements was made. The expected variance of retardance measurements on a particular double rhomb was estimated by the identification of the component of the variance σ_1^2 caused by random errors within these groups and the component of the variance σ_2^2 among the groups. We calculated σ_1^2 and σ_2^2 with the maximum-likelihood method for the components-of-variance model.[22] The total standard deviation of retardance measurements on a particular rhomb is then given by $\sigma_T = (\sigma_1^2 + \sigma_2^2)^{1/2}$. For each double rhomb, Table 2 lists the total number of measurements and the number of groups. A typical random uncertainty is seen from the 68 retardance measurements made on rhomb SR6. Those data were used to estimate the expected value of retardance and its standard deviation. The mean retardance was 90.079° with a total standard deviation of 0.033°.

6. Total Error of Polarimeter

Having found no significant sources of systematic error, the combined standard uncertainty u_c for our polarimetric method is estimated by the standard deviation of multiple measurements. A summary of the measurement results and uncertainties for the six double rhombs measured is shown in Table 2. We report uncertainty with a coverage factor of 2 (see Ref. 1). The resulting expanded uncertainty $U = 2u_c$ is shown in Table 2. The largest expanded uncertainty for the six double rhombs measured is ±0.067°.

7. Conclusions

For the measurement of retardance, we describe an automated polarimeter based on a rotating polarizer,

which is robust against imperfections of its components. The performance is optimum when we measure quarter-wave retardance. The precision and accuracy of this polarimeter have been demonstrated by multiple measurements on stable, nominally quarter-wave double-Fresnel-rhomb retarders and by comparison with other measurement techniques. For the six retarders measured, we report a maximum expanded uncertainty of 0.067°, which takes into account both random and systematic error sources.

Appendix A: Description of Fresnel Double-Rhomb Retarder

Our retardance measurements were made on Fresnel double-rhombs (two single Fresnel rhombs concatenated so as to have the output beam colinear with the input). The double rhomb was selected because of its stability against environmental perturbations, alignment errors, and wavelength variations. A Fresnel rhomb is a TIR device that causes light transmitted through it to experience a linear retardance resulting from the phase shift upon TIR. Light that passes through the double rhomb experiences four TIR's. Our rhombs were made from a lead-doped flint glass (847238) with a low stress-optic coefficient. The angles of the rhombs are cut so that the TIR's produce a total retardance of 90° (nominal). The clear aperture of the double rhomb is approximately 0.8 cm and its length approximately 15 cm. For a more detailed explanation of the double-rhomb geometry and performance, see Ref. 7.

References and Notes

1. The term expanded uncertainty refers to our multiplication of the measured uncertainty by a coverage factor of 2 to give an approximate 95% confidence interval. For more details, see B. N. Taylor and C. E. Kuyatt, eds. "Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results," Technical Note 1297, 1994 (National Institute of Standards and Technology, Gaithersburg, Md.)
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